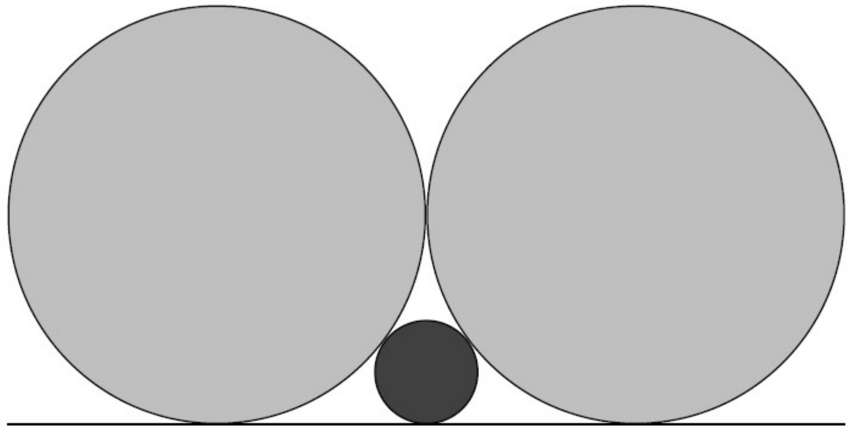

Problem 1: Three Circles

The three circles in the figure below touch each other pairwise and they all touch the bottom line. The two light gray discs each have an area of 25. What is the area of the dark gray disc?

Problem 2: Weight Watching

A huge bag contains infinitely many balls of different weights. For each non-negative integer $n \geq 0$, it contains 25 balls of weight n. Thus, there are 25 balls of weight 0, 25 balls of weight 1, 25 balls of weight 2, etc. Let us assume that days are numbered: $d = 0, 1, 2, 3, \ldots$

Is it possible to take out exactly five balls on each day d (until forever), such that the total weight of the balls taken out of the bag on day d is equal to d? For example, on day 1 the only way to do that is to pick four balls of weight 0 and one ball of weight 1.

If you think that this is possible, then show how. Otherwise, give an argument why this is not possible.

Problem 3: Line Dancing

Professor K makes a euphoric dance after making another scientific breakthrough. The dance consists of 25 equally sized steps, 14 to the left and 11 to the right. So, indeed professor K is performing a line dance. Obviously, at the end of his little dance Professor K is three steps to the left of his starting position.

a. How many different possibilities are there for professor K's dance?
b. How many of those dances start with a step to the right?
c. It turns out that, once he started his little dance, the professor never visited his starting position anymore. How many different possibilities are there now for professor K's dance?

Problem 4: Cards in Space

Lando bets his spaceship against Han in a party of the most famous card game in the universe. We shuffle 24 cards, numbered from 1 to 24, then we deal half of the cards to Lando and half of the cards to Han. One at a time, they play one of their cards on the table, with the number visible. The winner is the first player who plays a card such that the sum of the cards on the table is divisible by 25. If Lando starts, and if each player plays optimally, what is the probability that Han wins (and then becomes the new owner of the Falcon Millenium)?

Problem 5: ‘Goes out one ear and into the other’

Today is party time, as we celebrate the 25th edition of the MMM. My garden gnome participates in these festivities, and I have decorated it with a special red party hat, which carries the five-pointed star of the city of Maastricht that I placed right on the front, with its center exactly in the middle of the party hat.

The upper part of the garden gnome’s head (which for the most part fortunately is hidden under the hat) is shaped as a sphere, and the gnome’s two ear holes happen to be located precisely diametrically opposite each other on that sphere. When measured in centimeters, the distance between the two ear holes (the diameter of the sphere) exactly equals the square root of 136.

The party hat itself is shaped as a cone and it perfectly fits on the garden gnome’s head. To specify this further: it is tangent to that sphere just there where the brim of the hat touches it in a circle. This circle is of course a little smaller than the ‘great circles’ on the sphere: if the garden gnome stands straight up, then this touching circle is placed in a horizontal plane exactly 2 cm above the horizontal plane that passes through the two ear holes.

This morning I witnessed a surprising scene. An ant crawled out of the garden gnome’s left ear! It first crawled up to the brim of the party hat to the point (2 cm higher) that is straight above the left ear. I later measured that in doing so, it covered a distance of 2.51 cm. (Clearly, it needed a bit more than 2 cm to get there, because as you can see the ears aren’t nicely shaped as a sphere.) Next, from that point on the brim onward, the ant crawled further across the party hat in the shortest way possible to the center of the five-pointed star of Maastricht. Then it continued its way to the right ear, in which it finally disappeared, by following a path which precisely mirrored the path it took on the left side of the garden gnome’s head and hat.

Question (a): My garden gnome is English, and it’s indicated on the label that without the hat it’s exactly 20.60 inches tall. How tall is it, in inches, now that it is wearing the hat?

Question (b): What is the total distance covered by the ant, in centimeters, when it went out one ear and into the other, along the path described above?

(Note: By definition, 1 inch exactly equals 2.54 cm. For both questions: specify your answers to 2 decimals after the decimal point.)