Maastricht University School of Business and Economics
Problem Based Learning (PBL) Case
BSc Econometrics & Operations Research

The travelling student problem

After successfully finishing her first year of studies in Maastricht, Sandra is planning her summer vacation. She wants to visit the eleven largest cities in the Netherlands starting from Maastricht. In the end she wants to return to Maastricht.

Can you help her find a route on which she spends as little time as possible in the car?

This is what is commonly known as the *travelling salesperson problem* in Mathematics & Computer Science and one of the most famous problems in Operation Research. It originates in the 19th century and many new algorithmic ideas and techniques have been developed for and tested on instances of this problem. In fact, it is still unknown whether this problem can be solved efficiently for every possible input – a solution would solve one of the seven Millennium Problems which have been selected by the Clay Mathematics Institute in the year 2000. Only one of those has been solved and for the solution of each of the remaining ones a prize of US$ 1,000,000 will be awarded.

For this case you will explore some ideas how the problem of Sandra travelling through the Netherlands can be approached.

Below you see the distances between the twelve cities that Sandra is going to visit.

Distance (in km)		Maastricht	Amsterdam	Rotterdam	The Hague	Utrecht	Eindhoven	Tilburg	Almere	Groningen	Breda	Nijmegen	Apeldoorn
1	Maastricht		211	198	232	179	90	121	220	336	143	137	198
2	Amsterdam	211		79	63	47	124	111	26	179	103	119	87
3	Rotterdam	198	79		29	61	111	82	105	246	53	113	127
4	The Hague	232	63	29		66	138	113	80	232	84	140	132
5	Utrecht	179	47	61	66		85	79	45	187	71	85	68
6	Eindhoven	90	124	111	138	85		32	126	251	55	68	114
7	Tilburg	121	111	82	113	79	32		119	254	26	69	117
8	Almere	220	26	105	80	45	126	119		161	111	111	82
9	Groningen	336	179	246	232	187	251	254	161		253	203	142
10	Breda	143	103	53	84	71	55	26	111	253		95	132
11	Nijmegen	137	119	113	140	85	68	69	111	203	95		58
12	Apeldoorn	198	87	127	132	68	114	117	82	142	132	58	

During the tutorial meeting, please discuss your answers and ideas to the following five questions:

1. The first method you should use to find a route is the “Random Route”. Get out two dice to do this.
   - Start the route in Maastricht.
   - Roll both dice. The next city to be visited is the one whose number in the table corresponds to the sum of the two dice values. If the city has been visited already, roll the dice again.
   - Once all cities have been visited, return to Maastricht.

   Calculate the total distance of your route.
2. Now you should try something smarter, the “Nearest Neighbor Rule”.
   - Start the route in Maastricht.
   - Visit the city that is closest to Maastricht. Then keep going like this: visit the city that is closest to the current city but has not been visited yet.
   - Once all cities have been visited, return to Maastricht.

   Again, calculate the total distance of your route.
3. Can you find a route that has an even shorter total distance than one of the two previous ones?
4. One potential way to find the optimal solution to this problem would be the “brute force approach”, i.e. to consider all possible routes and choose the one with the shortest distance. Can you determine how many possible routes there are? If it takes a computer one millisecond to calculate the distance of one route, how long would that approach take?
5. Which companies might face a problem similar to this one? Which other real-life aspects of their problem would have to be taken into account and what are potential other objectives than just the total distance of the tour?